Towards Trustworthy Autonomy: Reliable and Efficient Interval Estimation and Learning for Robust Model Predictive Control [*]

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Abstract

This paper proposes a reliable and efficient interval estimation and learning framework (InterGP) for controlling systems under environmental and model uncertainties. As a preliminary result, we present a novel interval estimation design for system states and large uncertainties. The proposed interval estimation algorithm performs as well as set-membership methods with smaller computation power and produces a tighter bound compared to classic interval observers. Additionally, we discuss how to design the learning part in InterGP and leverage its advantage with robust control design.

Introduction

Model predictive control (MPC), a finite-horizon optimal control method, gains a lot of attention due to its superior performance and capability of handling constraints in various autonomy control problems such as platooning (Kianfar, Falcone, and Fredriksson 2015; Zheng et al. 2016), aggressive driving (Drews et al. 2017), and path planning (Rasekhipour et al. 2016; Ji et al. 2016; Kim et al. 2021). One of the fundamental problems of the MPC is that it can perform poorly and cannot guarantee (safety) constraints in the presence of dynamic and environmental uncertainties. To deal with systems subject to uncertainty with performance guarantees, robust MPC has been studied, such as min-max MPC (Lu and Arkun 2000) and tube MPC (Mayne et al. 2011; Lopez, Slotine, and How 2019). However, the current robust control approaches are ideal for handling small uncertainties, i.e., they can only ensure the performance if the true system is in the neighborhood of the nominal system. It is essential to develop a method to robustly control systems with large uncertainties induced by external disturbances and modeling errors.

Problem Setup

Consider the discrete-time system with environmental and model uncertainties:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \underbrace{\boldsymbol{A}_k\boldsymbol{x}_k + \boldsymbol{B}_k\boldsymbol{u}_k}_{\textit{a priori model}} + \underbrace{f(\boldsymbol{x}_k,\boldsymbol{u}_k)}_{\textit{model uncertainty}} + \boldsymbol{\omega}_k \\ \boldsymbol{y}_k &= \boldsymbol{C}_k\boldsymbol{x}_k + \boldsymbol{\nu}_k, \end{aligned} \tag{1}$$

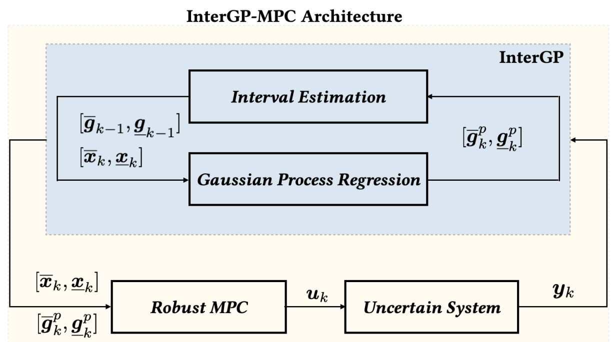

Figure 1: InterGP-MPC architecture.

where $\boldsymbol{x}_k \in \mathcal{X} \subseteq \mathbb{R}^n$, $\boldsymbol{u}_k \in \mathcal{U} \subseteq \mathbb{R}^p$ and $\boldsymbol{y}_k \in \mathbb{R}^m$ are the system state, the control input and the system output, respectively. The matrices $\boldsymbol{A}_k$, $\boldsymbol{B}_k$ and $\boldsymbol{C}_k$ are known. The *unknown* nonlinear function $f : \mathcal{X} \times \mathcal{U} \to \mathcal{F} \subseteq \mathbb{R}^n$ represents the model uncertainty. Noises $\boldsymbol{\omega}_k \in \mathbb{R}^n$ and $\boldsymbol{\nu}_k \in \mathbb{R}^m$ are unknown, but they are element-wise bounded by the known constant vectors, i.e. $\underline{\boldsymbol{w}} \leq \boldsymbol{\omega}_k \leq \overline{\boldsymbol{w}}$ and $\underline{\boldsymbol{v}} \leq \boldsymbol{\nu}_k \leq \overline{\boldsymbol{v}}$. Note that the we have no further assumptions on $\mathcal{F}$, which allows $\mathcal{F}$ to represent arbitrarily large model uncertainty.

Proposed Approach

We propose an InterGP-MPC architecture to control systems with environmental uncertainties and large model uncertainties. The proposed architecture is presented in Figure 1, where the interval estimation provides instantaneous bounded estimates for the system states and model uncertainties. The Gaussian process regression (GPR) provides the bounded functions for the model uncertainty, which are used as prior information for the interval estimation. Lastly, robust MPC utilizes the interval estimates and the bounded functions for the model uncertainty to control the system. Guaranteed uncertainty bounds with their quantified bound quality provided by InterGP ensure the reliability of the overall architecture; robust MPC guarantees robustness; and the efficiently designed interval estimation reduces overall computation cost. The current paper presents partial results of the proposed architecture. In particular, we focus on reliable and efficient interval estimation for the states and the

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model uncertainties. Then, we discuss how to design the learning part in InterGP and leverage its advantage with robust MPC design. The proposed approach has the following attractive properties:

- *Generality.* The proposed method can be applied to a wide class of systems since the large uncertainty setup. The priori model required in (1) can be a rough model. In addition, the proposed InterGP works with any other robust control approaches, while the proposed architecture utilizes robust MPC to control the uncertain system.
- *Reliability.* The real-world system suffers from disturbances and noises. As a consequence, the data for learning procedures are subject to uncertainty. However, most of the existing learning algorithms are designed under the assumption that the data are trustworthy. The proposed approach can address this issue by propagating the bounds of the state estimates in the InterGP cycle.
- *Decoupled design for learning and control.* Learning errors can destabilize the system, and unexpected system behaviors can affect the performance of estimation and learning. Using the proposed interval estimation of the states and the model uncertainties, we decouple the learning and control to eliminate the vicious circle between learning performance and the robustness of the control.
- *Efficiency.* Set-membership methods such as Combastel (2015); Tang et al. (2019) recursively track all possible intervals of states, but they require a high computation power. Classic interval observers (Efimov and Raïssi 2016) are computationally efficient, but they poorly perform. The proposed interval estimation algorithm performs as well as set-membership methods with a lower computation cost by solving trivial linear programmings (LP). Next, we review some concepts of *positive system*, which is critical to the interval estimation algorithm design.

Positive System

Consider the following linear discrete time system

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{A}_k \boldsymbol{x}_k + \boldsymbol{B}_k \boldsymbol{u}_k \\ \boldsymbol{y}_k &= \boldsymbol{C}_k \boldsymbol{x}_k, \end{aligned} \tag{2}$$

where $\boldsymbol{x}_k$, $\boldsymbol{u}_k$ and $\boldsymbol{y}_k$ are the state, control input and system output, respectively.

Definition 1 (Positive system) *System (2) is a positive system if for every positive initial condition and control input, i.e. $\boldsymbol{x}_0 \geq 0$ and $\boldsymbol{u}_0 \geq 0$, the state and the system output are positive, i.e. $\boldsymbol{x}_k \geq 0$ and $\boldsymbol{y}_k \geq 0\ \forall k \in \mathbb{Z}^+$.*

Definition 2 (Nonnegative matrix) *Matrix $\boldsymbol{A}$ is a nonnegative matrix if all of its elements a_{ij} are equal to or greater than zero, i.e. $a_{ij} \geq 0\ \forall i, j$.*

Theorem 1 (Theorem 2.6 in Kaczorek (2012)) *System (2) is a positive system if $\boldsymbol{A}_k$, $\boldsymbol{B}_k$, $\boldsymbol{C}_k$ and $\boldsymbol{D}_k$ are nonnegative matrices.*

It is worth emphasizing that the system in (1) does not need to be a positive system.

Notations. The matrix $\boldsymbol{I}$ denotes the identity matrix with an appropriate dimension. For a vector $\boldsymbol{a} \in \mathbb{R}^n$, we use $\overline{\boldsymbol{a}} \in \mathbb{R}^n$, and $\underline{\boldsymbol{a}} \in \mathbb{R}^n$ for element-wise estimates of upper bound and lower bound.

Preliminary Results

The current paper further assumes that the learning part (GPR in Figure 1) provides prior bounds of the model uncertainty $\overline{\boldsymbol{g}}^p_{k-1}$ and $\underline{\boldsymbol{g}}^p_{k-1}$.

Interval Estimation

The interval estimator tries to obtain an interval vector $[\overline{\boldsymbol{x}}_k, \underline{\boldsymbol{x}}_k]$ that contains $\boldsymbol{x}_k$ and an interval vector $[\overline{\boldsymbol{g}}_k, \underline{\boldsymbol{g}}_k]$ that contains $\boldsymbol{g}_k \triangleq |f(\boldsymbol{x}_k, \boldsymbol{u}_k)|$ for all $k \in \mathbb{Z}^+$, i.e.,

$$\underline{\boldsymbol{x}}_k \leq \boldsymbol{x}_k \leq \overline{\boldsymbol{x}}_k \textit{ and } \underline{\boldsymbol{g}}_k \leq \boldsymbol{g}_k \leq \overline{\boldsymbol{g}}_k$$

hold element-wisely. At time $k = 1$, we assume that the upper bound and the lower bound of the initial state estimates $\overline{\boldsymbol{x}}_0$ and $\underline{\boldsymbol{x}}_0$ are given. Via recursive estimation, we have previous bounds $\overline{\boldsymbol{x}}_{k-1}$ and $\underline{\boldsymbol{x}}_{k-1}$ at time k.

Model Uncertainty Estimation Given prior bounds of the model uncertainty $\overline{\boldsymbol{g}}^p_{k-1}$ and $\underline{\boldsymbol{g}}^p_{k-1}$, we can obtain the posterior bounds from output $\boldsymbol{y}_k$ by updating laws as follows:

$$\begin{aligned} \overline{\boldsymbol{g}}_{k-1} &= \overline{\boldsymbol{g}}^p_{k-1} + \overline{\boldsymbol{M}}_k \big(\boldsymbol{y}_k - \boldsymbol{C}_k \overline{\boldsymbol{x}}^p_k - \overline{\boldsymbol{v}}\big) \\ \underline{\boldsymbol{g}}_{k-1} &= \underline{\boldsymbol{g}}^p_{k-1} + \underline{\boldsymbol{M}}_k \big(\boldsymbol{y}_k - \boldsymbol{C}_k \underline{\boldsymbol{x}}^p_k - \underline{\boldsymbol{v}}\big), \end{aligned} \tag{3}$$

where

$$\begin{aligned} \overline{\boldsymbol{x}}^p_k &\triangleq \boldsymbol{A}_{k-1} \overline{\boldsymbol{x}}_{k-1} + \boldsymbol{B}_{k-1} \boldsymbol{u}_{k-1} + \overline{\boldsymbol{g}}^p_{k-1} + \overline{\boldsymbol{w}} \\ \underline{\boldsymbol{x}}^p_k &\triangleq \boldsymbol{A}_{k-1} \underline{\boldsymbol{x}}_{k-1} + \boldsymbol{B}_{k-1} \boldsymbol{u}_{k-1} + \underline{\boldsymbol{g}}^p_{k-1} + \underline{\boldsymbol{w}} \end{aligned} \tag{4}$$

are the prior bounds of the state. The gain matrices $\overline{\boldsymbol{M}}_k$ and $\underline{\boldsymbol{M}}_k$ in (3) will be selected later.

State Prediction and Estimation Given the posterior bounds of the model uncertainty $\overline{\boldsymbol{g}}_{k-1}$ and $\underline{\boldsymbol{g}}_{k-1}$, we predict the bounds of the state by

$$\begin{aligned} \overline{\boldsymbol{x}}^*_k &= \boldsymbol{A}_{k-1} \overline{\boldsymbol{x}}_{k-1} + \boldsymbol{B}_{k-1} \boldsymbol{u}_{k-1} + \overline{\boldsymbol{g}}_{k-1} + \overline{\boldsymbol{w}} \\ \underline{\boldsymbol{x}}^*_k &= \boldsymbol{A}_{k-1} \underline{\boldsymbol{x}}_{k-1} + \boldsymbol{B}_{k-1} \boldsymbol{u}_{k-1} + \underline{\boldsymbol{g}}_{k-1} + \underline{\boldsymbol{w}}. \end{aligned} \tag{5}$$

Then the state estimation is induced by utilizing the output $\boldsymbol{y}_k$ to correct the prediction in (5) as follows:

$$\begin{aligned} \overline{\boldsymbol{x}}_k &= \overline{\boldsymbol{x}}^*_k + \overline{\boldsymbol{L}}_k (\boldsymbol{y}_k - \boldsymbol{C}_k \overline{\boldsymbol{x}}^*_k - \overline{\boldsymbol{v}}) \\ \underline{\boldsymbol{x}}_k &= \underline{\boldsymbol{x}}^*_k + \underline{\boldsymbol{L}}_k (\boldsymbol{y}_k - \boldsymbol{C}_k \underline{\boldsymbol{x}}^*_k - \underline{\boldsymbol{v}}), \end{aligned} \tag{6}$$

where gain matrices $\overline{\boldsymbol{L}}_k$ and $\underline{\boldsymbol{L}}_k$ will be selected later.

Positive Error Dynamics To ensure the state and the uncertainty are bounded by (6) and (3) respectively is equivalent to requiring the estimation errors

$$\overline{\boldsymbol{e}}^x_k \triangleq \overline{\boldsymbol{x}}_k - \boldsymbol{x}_k, \qquad \underline{\boldsymbol{e}}^x_k \triangleq \boldsymbol{x}_k - \underline{\boldsymbol{x}}_k \tag{7}$$

and

$$\overline{\boldsymbol{e}}^g_k \triangleq \overline{\boldsymbol{g}}_k - \boldsymbol{g}_k, \qquad \underline{\boldsymbol{e}}^g_k \triangleq \boldsymbol{g}_k - \underline{\boldsymbol{g}}_k \tag{8}$$

to be nonnegative for $\forall k \in \mathbb{Z}^+$, i.e. to be positive systems. The following lemmas state the conditions such that the error dynamics are positive systems.

Assumption 1 *Let assume* $\overline{e}_0^x, \underline{e}_0^x \geq 0$, *and* $\underline{g}_k^p \leq g_k \leq \overline{g}_k^p$.

Lemma 1 (Uncertainty Estimation) *Consider Assumption 1, and assume that* $\overline{e}_k^x, \underline{e}_k^x \geq 0\ \forall k \in \mathbb{Z}^+$. *Then the uncertainty* g_{k-1} *is bounded by (3), i.e.*

$$\underline{g}_{k-1} \leq g_{k-1} \leq \overline{g}_{k-1} \tag{9}$$

for all $k \in \mathbb{Z}^+$, *if the following conditions hold*

$$I - \overline{M}_k C_k \geq 0 \tag{10}$$
$$I - \underline{M}_k C_k \geq 0 \tag{11}$$
$$-\overline{M}_k C_k A_{k-1} \geq 0 \tag{12}$$
$$-\underline{M}_k C_k A_{k-1} \geq 0 \tag{13}$$
$$-\overline{M}_k C_k \geq 0 \tag{14}$$
$$-\underline{M}_k C_k \geq 0 \tag{15}$$
$$-\overline{M}_k \geq 0 \tag{16}$$
$$-\underline{M}_k \geq 0. \tag{17}$$

Proof: To prove (9) is equivalent to showing that bounds of the error dynamics in (8) are nonnegative

$$\overline{e}_{k-1}^g \geq 0 \text{ and } \underline{e}_{k-1}^g \geq 0. \tag{18}$$

The upper bound of the error dynamics $\overline{e}_{k-1}^g$ can be described by

$$\begin{aligned}\overline{e}_{k-1}^g =& (I - \overline{M}_k C_k)\overline{e}_{k-1}^{gp} - \overline{M}_k C_k A_{k-1}\overline{e}_{k-1}^x \\ &- \overline{M}_k (C_k \overline{e}_{k-1}^w + \overline{e}_k^v),\end{aligned} \tag{19}$$

where $\overline{e}_{k-1}^{gp} \triangleq \overline{g}_{k-1}^p - g_{k-1}$, $\overline{e}_{k-1}^w \triangleq \overline{w} - \omega_{k-1}$, and $\overline{e}_k^v \triangleq \overline{v} - \nu_k$. Since the conditions (10), (12), (14) and (16) hold, all three terms in (19) are nonnegative, i.e. $\overline{e}_{k-1}^g \geq 0$. The statement $\underline{e}_{k-1}^g \geq 0$ can be proven by a similar procedure, which is omitted here. Therefore we have (18), which completes the proof. ■

Remark 1 *Lemma 1 holds under the assumption of the nonnegativity of bounds of the state error dynamics. The following Lemma 2 demonstrates the nonnegativity of* $\overline{e}_k^x$ *and* $\underline{e}_k^x$.

Lemma 2 (State Estimation) *Under Assumption 1, the ground truth of the state is bounded by (6), i.e.* $\underline{x}_k \leq x_k \leq \overline{x}_k\ \forall k \in \mathbb{Z}^+$, *if the following conditions hold*

$$(I - \overline{M}_k C_k)(I - \overline{L}_k C_k) A_{k-1} \geq 0 \tag{20}$$
$$(I - \underline{M}_k C_k)(I - \underline{L}_k C_k) A_{k-1} \geq 0 \tag{21}$$
$$(I - \overline{M}_k C_k)(I - \overline{L}_k C_k) \geq 0 \tag{22}$$
$$(I - \underline{M}_k C_k)(I - \underline{L}_k C_k) \geq 0 \tag{23}$$
$$-(I - \overline{M}_k C_k)\overline{L}_k \geq 0 \tag{24}$$
$$-(I - \underline{M}_k C_k)\underline{L}_k \geq 0 \tag{25}$$
$$-\overline{L}_k \geq 0 \tag{26}$$
$$-\underline{L}_k \geq 0. \tag{27}$$

Proof: The upper bound of the error dynamics $\overline{e}_k^x$ is given by

$$\begin{aligned}\overline{e}_k^x =& (I - \overline{L}_k C_k) A_{k-1}\overline{e}_{k-1}^x + (I - \overline{L}_k C_k)\overline{e}_{k-1}^g \\ &+ (I - \overline{L}_k C_k)\overline{e}_{k-1}^w - \overline{L}_k \overline{e}_k^v.\end{aligned} \tag{28}$$

Plugging (19) into (28), we have

$$\begin{aligned}\overline{e}_k^x =& (I - \overline{L}_k C_k)(I - \overline{M}_k C_k) A_{k-1}\overline{e}_{k-1}^x \\ &+ (I - \overline{L}_k C_k)(I - \overline{M}_k C_k)\overline{e}_{k-1}^{gp} \\ &+ (I - \overline{L}_k C_k)(I - \overline{M}_k C_k)\overline{e}_{k-1}^w \\ &- (I - \overline{L}_k C_k)\overline{M}_k \overline{e}_k^v - \overline{L}_k \overline{e}_k^v.\end{aligned} \tag{29}$$

We have $\overline{e}_k^x \geq 0$ since the conditions (20), (22), (24) and (26) hold. The statement $\underline{e}_k^x \geq 0$ can be proven by a similar procedure. Therefore we have $\underline{x}_k \leq x_k \leq \overline{x}_k\ \forall k \in \mathbb{Z}^+$. ■

To keep the error dynamics positive systems, gain matrices should be selected under the constraints stated in Lemmas 1 and 2. On top of this, gain matrices $\overline{M}_k$ and $\underline{M}_k$ are selected such that the upper bound is minimized and lower bound is maximized:

$$\begin{aligned}&\min_{\overline{M}_k \leq 0} \|\overline{g}_{k-1}\| \\ &\textit{subject to } (10), (12), (14)\end{aligned} \tag{30}$$

$$\begin{aligned}&\max_{\underline{M}_k \leq 0} \|\underline{g}_{k-1}\| \\ &\textit{subject to } (11), (13), (15).\end{aligned} \tag{31}$$

Likewise, given $\overline{M}_k$ and $\underline{M}_k$, the gain matrices $\overline{M}_k$ and $\underline{M}_k$ are selected such that the upper bound is minimized and lower bound is maximized:

$$\begin{aligned}&\min_{\overline{L}_k \leq 0} \|\overline{x}_k\| \\ &\textit{subject to } (20), (22), (24)\end{aligned} \tag{32}$$

$$\begin{aligned}&\max_{\underline{L}_k \leq 0} \|\underline{x}_k\| \\ &\textit{subject to } (21), (23), (25).\end{aligned} \tag{33}$$

Optimization problems (30), (31), (32), and (33) are linear programming (LP) problems, which can be solved efficiently.

Lemma 3 *Feasible sets for linear programming (LP) problems (30), (31), (32), and (33) are nonempty.*

We omit the proof of this lemma because it is trivial to show that a zero matrix is always in the feasible sets.

Remark 2 *Optimization problems (30), (31), (32), and (33) minimize the impact of external disturbances and the error propagation from the previous state estimate. The cost function is not unique. For example, we may choose* $\ell_\infty - \ell_\infty$ *observer which minimizes the impact of external disturbances* ω_k *and* ν_k *as in Briat and Khammash (2016).*

Remark 3 *Positive systems constitute a remarkable class of systems and receive increasing attention and appear frequently in practical applications (Kaczorek 2012; Shorten, Wirth, and Leith 2006; Farina and Rinaldi 2011). Notice that we only assume the error dynamics to be positive systems. The original system does not need to be a positive system. But if it is a positive system, the LP problems (30), (31), (32), and (33) can be further simplified.*

Algorithm 1: Interval estimation for uncertainty and state

Require: $\overline{\boldsymbol{g}}^p_{k-1}$, $\underline{\boldsymbol{g}}^p_{k-1}$, $\boldsymbol{y}_k$

▷ State priori

Obtain $\overline{\boldsymbol{M}}_k$ and $\underline{\boldsymbol{M}}_k$ by solving LP problems (30) and (31);

$[\overline{\boldsymbol{x}}^p_k, \underline{\boldsymbol{x}}^p_k] \leftarrow$ STATE PRIORI as in (4);

▷ Uncertainty posterior

$[\overline{\boldsymbol{g}}_{k-1}, \underline{\boldsymbol{g}}_{k-1}] \leftarrow$ UNCERTAINTY POSTERIOR as in (3);

▷ State prediction

$[\overline{\boldsymbol{x}}^*_k, \underline{\boldsymbol{x}}^*_k] \leftarrow$ STATE PREDICTION as in (5);

▷ State estimation

Obtain $\overline{\boldsymbol{L}}_k$ and $\underline{\boldsymbol{L}}_k$ by solving LP problems (32) and (33);

$[\overline{\boldsymbol{x}}_k, \underline{\boldsymbol{x}}_k] \leftarrow$ STATE ESTIMATION as in (6);

Simulation Example

This simulation study compares the performance and efficiency of three interval estimation algorithms, i.e. the proposed approach, interval observer (Efimov and Raïssi 2016), and set-membership method (Tang et al. 2019). Considering a DC servo motor described in Buciakowski et al. (2017), we use the same dynamic model and parameters used in Tang et al. (2019). The interval estimations of the motor speed n_{motor} by the aforementioned algorithms are shown in Figure 2. It shows that the proposed approach is more accurate than the interval observer and has similar accuracy as the set-membership method. In addition, the performance comparison including the running time and the averaged estimation width ($\frac{\sum \overline{n}_{motor} - \underline{n}_{motor}}{40}$) is provided in Table 1, which demonstrates the reliability and efficiency of the proposed approach.

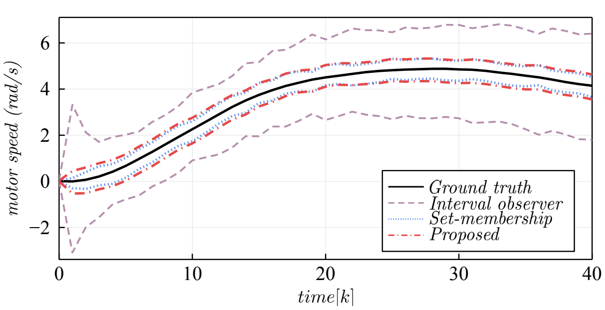

Figure 2: Comparison of interval estimation algorithms.

Table 1: Performance comparison.

Algorithm	Time (s)	Estimation width (rad/s)
Proposed	**0.880**	**1.003**
Interval observer	1.283	3.693
Set-membership	12.399	0.852

Future Work

We will develop the learning part in InterGP and implement robust MPC to complete the proposed architecture outlined in Figure 1.

Learning through GPR

We denote the input as $\mathbf{x}_k \triangleq [\overline{\boldsymbol{x}}_k, \underline{\boldsymbol{x}}_k, \boldsymbol{u}_k]$, and the output as $\mathbf{y}_k \triangleq [\overline{\boldsymbol{g}}_k, \underline{\boldsymbol{g}}_k]$, and construct the dataset as $\{(\mathbf{x}_i, \mathbf{y}_i) \mid i = 0, \cdots, k\}$. GPR (Rasmussen and Williams 2006) is a non-parametric regression algorithm that can be used to learn dynamic systems. Consider the model $\mathbf{y}_i = f(\mathbf{x}_i) + \varepsilon_i$. In GPR, we assume that function f is a Gaussian process, and the output $\mathbf{y}$ is corrupted by additive Gaussian noise $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. The mean function and covariance function can be found by

$$\mu(\mathbf{x}_*) = k_*^T (K(\mathbf{x}, \mathbf{x}) + \sigma^2 \boldsymbol{I})^{-1} \mathbf{y}$$
$$\Sigma(\mathbf{x}_*) = k(\mathbf{x}_*, \mathbf{x}_*) - k_*^T (K(\mathbf{x}, \mathbf{x}) + \sigma^2 \boldsymbol{I})^{-1} k_*, \quad (34)$$

where $k_* \triangleq [k(\mathbf{x}_*, \mathbf{x}_1), \cdots, k(\mathbf{x}_*, \mathbf{x}_p)]^\top$. One of the key challenge of constructing InterGP is to analyze how the learning variance in (34) affects the interval observer estimator.

Instead of using GPR, locally weighted regression (LWR) (Atkeson, Moore, and Schaal 1997) is an alternative for learning. LWR can be seen as a simplified version of the GPR. Two critical differences are that $i)$ LWR does not calculate sample covariance, and $ii)$ LWR does not consider the correlation between samples, while GPR utilizes the correlation between samples which is multi-variate Gaussian. Because of the first difference, GPR quantifies the quality of the regression as a covariance, while LWR does not. However, because of the second property, GPR needs to inverse $p \times p$ matrix and thus suffers from computational complexity $O(p^3)$, while LWR does not have such an issue.

InterGP and Robust MPC

The proposed InterGP in Figure 1 will be fed into robust MPC (Mayne et al. 2011; Lopez, Slotine, and How 2019). We use the guaranteed intervals with their learning quality (GP variance in (34)) to trading-off conservative and performance in robust MPC. As a result, the proposed control architecture is expected to be a reliable and robust control architecture with efficient estimation and learning in the presence of dynamic and environmental uncertainties.

Conclusion

The current paper proposed an InterGP-MPC architecture for systems with large uncertainties. The interval estimation provides the bound of the model uncertainty, and GPR collects these estimates to learn the uncertainty function. Robust MPC uses the interval estimates and the bound functions for the unknown dynamics to control the system robustly. We developed an efficient interval estimation for system states and uncertainty and empirically showed the performance of the proposed interval estimation algorithm by comparing it to others. We will combine the developed interval estimation with GP and MPC to achieve reliable and efficient robust control architecture.

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